The Gyrokinetic Regime
Geometry
Velocity Space
Linear How-To
**Units**
Non-Linear Issues
Miscellany
Units

- GS2 started as a theory code, and so works exclusively with normalized quantities.

- The user must decide what the ‘reference’ units will be for charge, mass, density, temperature, magnetic field, and equilibrium-scale lengths.

- User must also choose a normalization convention for the thermal velocity, either \( v_t = \sqrt{2T/m} \) or \( v_t = \sqrt{T/m} \).

- Choice of units determines conversion factors for input and output.
**Recommended Units** \( \text{species\_parameters} \_i \)

Charge: Input charge \( Z \) in multiples of \(|e|\). *I.e.*, set electron charge \( Z = -1 \), and so on.

Mass: Input \textit{mass} in units of the AMU of the dominant ion species.

Density: Input \textit{dens} in units of electron density.

Temp: Input \textit{temp} in units of the temperature of a species of your choice, typically the dominant ions or the electrons.

Given these definitions and a field, can define \( \rho_r \) and \( v_{tr} \), the gyroradius and thermal speed of the reference species.
**Recommended Normalizations**

**Length:** Normalize equilibrium-scale lengths to either the major radius ("theory" applications) or the minor radius ("experimental" applications); call this quantity $a^*$ here.

**Field:** Magnetic field normalization depends on geometry assumptions.

(a) Shifted-circles or local equilibrium: Use $B_a$

(b) Numerical equilibria: Normalizing field comes from file, written to eik2.out.
Gradients

- Density and temperature gradient scale length definitions depend on definition of flux-surface label $\rho$ and choice of normalizing length.

- Consider the $\omega_*$ term from the GK equation in detail.

\[
-i \frac{B_0 \times \nabla S \cdot \nabla F_0}{B_0 m \Omega} q \hat{\chi} = -i n_0 \frac{c}{B_0} \hat{\chi} [\hat{b} \times \nabla (\alpha + q \theta_0) \cdot \nabla F_0]
\]

where

\[
\hat{\chi} = \left(\phi - \frac{v_{||}}{c} \psi\right) J_0 + \frac{\hat{\sigma} |\nabla S| v_{\perp}}{c} J_1.
\]
Origin of Normalizations

- Use $F_0 = F_0(\psi)$ for spatial dependence:

$$-in_0 \frac{c}{B_0} \tilde{\chi} [\hat{b} \times \nabla (\alpha + q\theta_0) \cdot \nabla F_0] = -in_0 \frac{c}{B_0} \tilde{\chi} (\hat{b} \cdot \nabla \alpha \times \nabla \psi) \frac{\partial F_0}{\partial \psi}$$

$$= -in_0 c \tilde{\chi} \frac{\partial F_0}{\partial \psi} = -i \left( \frac{n_0}{a} \frac{d\rho}{d\psi_N} \right) \left( \frac{d\psi_N}{d\psi} \right) c \tilde{\chi} \frac{\partial F_0}{\partial \rho}$$

- Decision: Scale perturbed fields according to

$$\chi = \frac{|e| \tilde{\chi}_{\text{phys}}}{T_r} \left( \frac{a_*}{\rho_r} \right)$$

where the values of $e$ and $a$ are determined by the choice of units.
Effect of Normalizations

- Recalling known normalization of $\Psi$, can write this as

$$-i\left(\frac{n_0}{a} \frac{d\rho}{d\psi_N}\right) \left(\frac{d\psi_N}{d\psi}\right) c \hat{\chi} \frac{\partial F_0}{\partial \rho} = -i k_\theta \frac{cT}{e B a a a_*} \rho_r \chi \frac{\partial F_0}{\partial \rho} = -i k_\theta \rho_r \frac{\rho v_{tr}}{a_*^2} \frac{a}{\partial \rho} \chi$$

- GK equation divided through by $F_0$, and factor in red scales out, leaving a term like:

$$i(k_\theta \rho_r) a^* \frac{1}{a} \frac{\partial F_0}{\partial \rho} \chi$$

- The derivative of $F_0$ here involves the appropriately normalized temperature and density gradient scale lengths.
Gradient Normalizations \textit{species} \_\textit{parameters} \_\textit{i}

- Consequences of foregoing:

\[
\begin{align*}
\text{fprim} &= \frac{a_* 1}{a} \frac{dn_0}{d\rho} \\
\text{tprim} &= \frac{a_* 1}{a} \frac{dT_0}{d\rho}
\end{align*}
\]

- If \( a_* = R \), then these are expressions like \( R/L \); otherwise, \( a/L \).

- In either case, the choice of \( \rho \) is important and potentially confusing. \textbf{Be careful}. \texttt{Do \_prep} and \texttt{TRXPL} are very helpful at this point.
Further Implications

- All perturbed quantities are normalized like the fields were:
  \[ n_{\text{sim}} = \frac{\tilde{n}_{\text{phys}}}{n_r} \left( \frac{a_*}{\rho_r} \right), \quad T_{\text{sim}} = \frac{\tilde{T}_{\text{phys}}}{T_r} \left( \frac{a_*}{\rho_r} \right), \quad \text{etc.} \]

- These are the necessary conversion factors for getting physical fluctuation amplitudes from the simulation data.

- Fluxes have similar conversion factors, described next.

- Caution: Quantities broken down by mode (e.g., phi2_by_k) have additional factors related to the Fourier transform conventions.
Heat Flux Normalizations

- Recall that the fluxes are reported in a way that effectively separates the choice of $\rho$ for a given application from the choice made for a GS2 calculation (through use of area factor $A$).

- Rest of normalization is:

$$Q_{\text{phys}} = Q_{\text{sim}} \left( \frac{\rho_r}{a_*} \right)^2 v_{tr} n_r k_b T_r,$$

in which I have assumed that the reference temperature is in energy units (keV, eV, whatever) so that the Boltzmann constant is needed.
Particle Flux Normalizations

- Similarly, for particle fluxes:

$$\Gamma_{phys} = \Gamma_{sim} \left( \frac{\rho_r}{a_*} \right)^2 n_r v_{tr}$$

- Note in particular that the reported fluxes by species are in proportion; there are no species-specific factors required to convert a reported flux to a physical flux.
Fourier Transform Conventions

- Will write this out explicitly later in this meeting. Results in factors of 2 here and there when one looks at data broken down by Fourier mode.