Units

- GS2 started as a theory code, and so works exclusively with normalized quantities.

- The user must decide what the ‘reference’ units will be for charge, mass, density, temperature, magnetic field, and equilibrium-scale lengths

- User must also choose a normalization convention for the thermal velocity, either $v_t = \sqrt{2T/m}$ or $v_t = \sqrt{T/m}$.

- Choice of units determines conversion factors for input and output

Recommended Units species_parameters_i

arge: Input charge Z in multiples of $|e|$. *I.e.,* set electron charge $Z = -1$, and so on.

Mass: Input mass in units of the AMU of the dominant ion species.

nsity: Input dens in units of electron density.

Temp: Input temp in units of the temperature of a species of your choice, typically the dominant ions or the electrons.

Given these definitions and a field, can define ρ_r and v_{tr}, the gyroradius and thermal speed of the reference species.

Recommended Normalizations

ngth: Normalize equilibrium-scale lengths to either the major radius ("theory" applications) or the minor radius ("experimental" applications); call this quantity $a*$ here.

field: Magnetic field normalization depends on geometry assumptions.

(a) Shifted-circles or local equilibrium: Use B_a

(b) Numerical equilibria: Normalizing field comes from file, written to `eik2.out`.

Gradients

- Density and temperature gradient scale length definitions depend on definition of flux-surface label ρ *and* choice of normalizing length.

- Consider the ω_* term from the GK equation in detail.

$$-i\frac{\mathbf{B_0} \times \nabla S \cdot \nabla F_0}{B_0 m \Omega} q\widehat{\chi} = -in_0 \frac{c}{B_0} \widehat{\chi} \left[\hat{\mathbf{b}} \times \nabla \left(\alpha + q\theta_0 \right) \cdot \nabla F_0 \right]$$

where

$$\widehat{\chi} = \left(\widehat{\phi} - \frac{v_\parallel}{c} \widehat{\psi} \right) J_0 + \frac{\widehat{\sigma} |\nabla S| v_\perp}{c} J_1.$$

Origin of Normalizations

- Use $F_0 = F_0(\Psi)$ for spatial dependence:

$$-in_0 \frac{c}{B_0} \widehat{\chi} \left[\hat{\mathbf{b}} \times \nabla \left(\alpha + q\theta_0 \right) \cdot \nabla F_0 \right] = -in_0 \frac{c}{B_0} \widehat{\chi} \left(\hat{\mathbf{b}} \cdot \nabla \alpha \times \nabla \Psi \right) \frac{\partial F_0}{\partial \Psi}$$

$$= -in_0 c \widehat{\chi} \frac{\partial F_0}{\partial \Psi} = -i \left(\frac{n_0}{a} \frac{d\rho}{d\Psi_N} \right) \left(\frac{d\Psi_N}{d\Psi} \right) c \widehat{\chi} \frac{\partial F_0}{\partial \rho}$$

- Decision: Scale perturbed fields according to

$$\chi = \frac{|e| \widehat{\chi}_{\mathrm{phys}}}{T_r} \left(\frac{a_*}{\rho_r} \right)$$

where the values of e and a are determined by the choice of units.

Effect of Normalizations

- Recalling known normalization of Ψ, can write this as

$$-i\left(\frac{n_0}{a}\frac{d\rho}{d\Psi_N}\right)\left(\frac{d\Psi_N}{d\Psi}\right)c\hat{\chi}\frac{\partial F_0}{\partial\rho} = -ik_\theta\frac{cT}{eB_a}\frac{\rho_r}{aa_*}\chi\frac{\partial F_0}{\partial\rho} = -ik_\theta\rho_r\frac{\rho_r v_{tr}}{a_*^2}\frac{a_*}{a}\frac{\partial F_0}{\partial\rho}\chi$$

- GK equation divided through by F_0, and factor in red scales out, leaving a term like:

$$i(k_\theta\rho_r)\frac{a_*}{a}\frac{1}{F_0}\frac{\partial F_0}{\partial\rho}\chi$$

- The derivative of F_0 here involves the appropriately normalized temperature and density gradient scale lengths.

Gradient Normalizations `species_parameters_i`

- Consequences of foregoing:

$$\texttt{fprim} = \frac{a_*}{a}\frac{1}{n_0}\frac{dn_0}{d\rho} \qquad\qquad \texttt{tprim} = \frac{a_*}{a}\frac{1}{T_0}\frac{dT_0}{d\rho}$$

- If $a_* = R$, then these are expressions like R/L; otherwise, a/L.

- In either case, the choice of ρ is important and potentially confusing. Be careful. `Do_prep` and `TRXPL` are very helpful at this point.

Further Implications

- All perturbed quantities are normalized like the fields were:

$$n_{\mathsf{sim}} = \frac{\widehat{n}_{\mathsf{phys}}}{n_r}\left(\frac{a_*}{\rho_r}\right), \qquad T_{\mathsf{sim}} = \frac{\widehat{T}_{\mathsf{phys}}}{T_r}\left(\frac{a_*}{\rho_r}\right), \qquad etc.$$

- These are the necessary conversion factors for getting physical fluctuation amplitudes from the simulation data.

- Fluxes have similar conversion factors, described next.

- Caution: Quantities broken down by mode (*e.g.*, `phi2_by_k`) have additional factors related to the Fourier transform conventions.

Heat Flux Normalizations

- Recall that the fluxes are reported in a way that effectively separates the choice of ρ for a given application from the choice made for a `GS2` calculation (through use of area factor A).

- Rest of normalization is:

$$Q_{\mathsf{phys}} = Q_{\mathsf{sim}} \left(\frac{\rho_r}{a_*}\right)^2 v_{tr} n_r k_b T_r,$$

  in which I have assumed that the reference temperature is in energy units (keV, eV, whatever) so that the Boltzmann constant is needed.

Particle Flux Normalizations

- Similarly, for particle fluxes:

$$\Gamma_{\mathsf{phys}} = \Gamma_{\mathsf{sim}} \left(\frac{\rho_r}{a_*}\right)^2 n_r v_{tr}$$

- Note in particular that the reported fluxes by species are in proportion; there are no species-specific factors required to convert a reported flux to a physical flux.

Fourier Transform Conventions

- Will write this out explicitly later in this meeting. Results in factors of 2 here and there when one looks at data broken down by Fourier mode.