Phys 161: Notes

These notes are not complete! Use them in conjunction with the textbook, the lectures, discussion sections, study groups and the website.

Rotational Kinematics

• Circular motion again!

• A particle moving through an angle $\theta$ along a circular path of radius $r$ traverses a distance $s = r \theta$. (Remember that $\theta$ must be measured in degrees for this formula to work.) This is a geometric fact – not really physics at all.

• If we consider a particle moving along a circular path with some time-varying position given by $\theta(t)$, then we can use calculus together with the fact above to find the instantaneous speed:

$$v = \frac{ds}{dt} = r \frac{d\theta}{dt} \equiv r \omega.$$ 

In the last step, we are simply defining $\omega$, the instantaneous angular speed.

• A particle moving along a circular path of radius $r$ with constant speed $v$ has an angular speed of $\omega = v/r \text{ rad/s}$. Put another way, the speed of a particle moving along a circular path of radius $r$ with angular speed $\omega$ is $v = r \omega$.

• We can take another derivative of the position $s$ to find

$$a = \frac{d^2 s}{dt^2} = r \frac{d^2 \theta}{dt^2} \equiv r \alpha.$$ 

Again, the last step defines $\alpha$, the instantaneous angular acceleration.

• All we have done so far is use geometry, calculus and the definitions of velocity and acceleration.

• If the angular acceleration is constant, we can use the same techniques used in the beginning of the class to integrate

$$\frac{d^2 \theta}{dt^2} = \alpha_0$$

twice to find

$$\omega_f = \omega_0 + \alpha_0 t$$

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_0 t^2$$

and then rearrange these to find

$$\omega_f^2 = \omega_0^2 + 2 \alpha (\theta_f - \theta_0).$$
We introduce these new variables and equations because it turns out to be much easier to work out the physics of rigidly rotating objects using $\theta$, $\omega$ and $\alpha$ than the corresponding linear quantities. This is mainly because when a body rotates rigidly, any two points in the body are rotating with different velocities and different accelerations (unless they happen to lie at exactly the same distance from the axis of rotation).